SUMMARY SYLLABUS FOR THE RE-EXAM IN MATH 001 (New Stream)

CHAPTER 1 – REAL NUMBERS

The Natural Numbers: The numbers 1, 2, 3, 4, 5, … are called the natural numbers (counting numbers). The group of all natural numbers is denoted by **N.**

Whole numbers: The collection of all natural numbers along with zero is known as the set of whole numbers. They are 0, 1, 2, 3, …. It is denoted by **W.**

Integers: The set of all integers is denoted by **Z.** The set of integers include the natural numbers, the number zero and the negatives of the natural numbers. Thus, they are

…, -3, -2, -1, 0, 1, 2, 3, ….

Rational Numbers: The numbers which can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are called rational numbers. For eg, $\frac{22}{7}, \frac{-3}{5}, \frac{1}{-4}, \frac{-6}{-11}$, -3, 0, 8 etc. are rational numbers. Any terminating or non-terminating recurring decimal number is also a rational number. i.e, Numbers such as 1.75, 2.333…, 0. 2134134134… are rational numbers. The group of rational numbers is denoted by **Q.**

Operations on rational Numbers

Addition of Rational numbers : The sum of two rational numbers is always a rational number. To add two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we use the following rule:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

Subtraction of Rational numbers : The difference of two rational numbers is again a rational number. In order to subtract one rational number from another, we proceed in the same way as that of addition.

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

Multiplication of Rational numbers: The product of two rational numbers is another rational number whose numerator is the product of the numerators and the denominator is the product of the denominators.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Division of Rational numbers: The ratio of two non-zero rational numbers is again a rational number. To divide a rational number by another rational number, we multiply the first one by the reciprocal of the second.

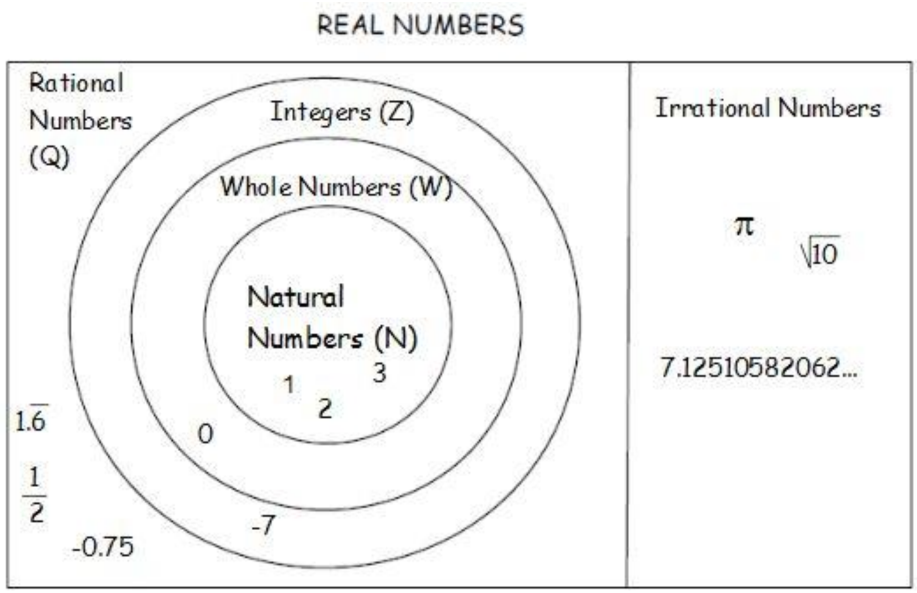
$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Irrational Numbers: Irrational numbers are numbers that cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Eg: $\sqrt{2}$, $\frac{1}{\sqrt{3}}$, 4+ π .. etc.

When written in the decimal form, irrational numbers are non-terminating and non-recurring. Thus 4.12432632… is an irrational number.

Note: The value of π is 3.14152965… Since it is approximately equal to $\frac{22}{7}$, we may write $\pi \approx \frac{22}{7}$.

Real Numbers: Real numbers consist of both rational and irrational numbers. The set of all real numbers is denoted by **R.** It is very much evident that the sum, difference, product and quotient of any two non-zero real numbers are real numbers. The following illustrates the family of real numbers.



***Example* 1:** Evaluate: $\frac{3}{6}+\frac{2}{8}$

Solution

$$\frac{3}{6}+\frac{2}{8}=\frac{3\times 8+2\times 6}{6\times 8}=\frac{24+12}{48} = \frac{36}{48}\div\frac{3}{3}$$

$$=\frac{12}{16}\div\frac{4}{4} = \frac{3}{4}$$

Example 2: Evaluate: $\frac{-3}{5}-\frac{2}{7}$

Solution

$$\frac{-3}{5}-\frac{2}{7}=\frac{-3\times 7-2\times 5}{5\times 7} = \frac{-21-10}{35} = \frac{-31}{35}$$

***Example 3*:** Evaluate: $\frac{5}{-2}\times\frac{-3}{10}\times\frac{1}{3}$

Solution

$$\frac{5}{-2}\times\frac{-3}{10}\times\frac{1}{3}=\frac{5\times(-3)\times 1}{(-2)\times 10\times 3}=\frac{-15}{-60}=\frac{1}{4}$$

CHAPTER 2 - EXPONENTS

Laws of Exponents

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$ or $\frac{a^m}{a^n}=a^{m-n}$
3. $\left(a^m\right)^n = a^{mn}$
4. $(ab)^m = a^m b^m$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
6. $a^{-n}=\frac{1}{a^n}$ In particular, $a^{-1}=\frac{1}{a}$
7. $\left(\frac{a}{b}\right)^{-n}=\frac{a^{-n}}{b^{-n}}=\frac{b^n}{a^n}=\left(\frac{b}{a}\right)^n$
8. For any non-zero real number a, $a^0 = 1$

***Example* 1:** Simplify the following:

i) $\left(a^3b^4\right)\left(a^2b\right)$ ii) $\frac{2^9}{2^4}$

iii) $\left(x^2y^3\right)^4$ iv) $\frac{24xyz^3}{-3z^2}$

Solution

i) $\left(a^3b^4\right)\left(a^2b\right) = a^{3+2}b^{4+1} = a^5b^5$

ii) $\frac{2^9}{2^4} = 2^{9-4} = 2^5 = 32$

iii) $(x^2y^3)^4 = (x^2)^4(y^3)^4 = x^{2\times 4}y^{3\times 4} = x^8y^{12}$

iv) $\frac{24xyz^3}{-3z^2} = -8xyz$

***Example* 2:** Show that $\left(a^xb^y\right)\left(\frac{b^{2x}}{a^{-y}}\right) = a^{x+y}b^{y+2x}$

Solution

$$\left(a^xb^y\right)\left(\frac{b^{2x}}{a^{-y}}\right) = a^xb^y.a^yb^{2x}$$

$$= a^xa^yb^yb^{2x}$$

$$= a^{x+y}b^{y+2x}$$

***Example* 3:** Simplify : $\left(4^{-1}+8^{-1}\right)\div\left(\frac{2}{3}\right)^{-1}$

Solution

$$\left(4^{-1}+8^{-1}\right)\div\left(\frac{2}{3}\right)^{-1} = \left(\frac{1}{4}+\frac{1}{8}\right)\div\left(\frac{3}{2}\right)$$

$$= \left(\frac{2}{8}+\frac{1}{8}\right)\div\left(\frac{3}{2}\right)$$

$$= \frac{3}{8}\div\frac{3}{2} = \frac{3}{8}\times\frac{2}{3} = \frac{1}{4}$$

***Example* 4:** Evaluate $\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\}\div\left(\frac{1}{4}\right)^{-3}$

Solution

$$\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\}\div\left(\frac{1}{4}\right)^{-3} = \left\{3^3-2^3\right\}\div 4^3 = \left\{27-8\right\}\div 64 = 19\div 64 = \frac{19}{64}$$

CHAPTER 3 - ALGEBRAIC EXPRESSIONS

3.1 Algebraic expressions

In algebra, we come across two types of quantities, namely constants and variables. A symbol having a fixed numerical value is called a *constant* and a symbol which takes various numerical values is known as a *variable*.

Definition of an Algebraic Expression : A combination of constants and variables connected by some or the entire four fundamental operations +, – , × and ÷ is called an **algebraic expression**.

Eg: $2x^2 - 3x + 5$, $\sqrt{x} + 6$

Terms: The different parts of the algebraic expression separated by the sign + or – are called the **terms** of the expression. The number present in each term is called the *numerical* **coefficient** *(or* **the coefficient***)*. A term without any variables is called the **constant term**.

Eg: (i) $5 - 3x + 4x^2y$ is an algebraic expression consisting of three terms, namely 5, $-3x$ and $4x^2y$. Here the constant term is 5 and the coefficient of x is -3.

(ii) $7x^2 - 5xy + y^2z - 8$ is an algebraic expression consists of four terms, namely $7x^2$, $-5xy$, y^2z and – 8. Here the constant term is -8 and the coefficient of y^2z is 1.

(iii) $3ab$ is an algebraic expression consists of one term, namely $3ab$. Here the coefficient of ab is 3.

Like and Unlike terms of an Algebraic Expression : Two or more terms of an algebraic expression are said to be **like terms** if

(i) they have the same variables and

(ii) the exponents (powers) of each variable are the same.

Otherwise, they are called *unlike terms.*

Eg: (a) $-a$, $7a$ and $5a$ are like terms

(b) The terms $4a^2b^3$, $7b^3a^2$ and $5a^2b^3$ are all like terms

(c) The terms x^3y^2 and x^2y^3 are unlike as the powers of the variables are different.

Operations of Algebraic Expressions : The sum of several like terms is another like term whose coefficient is the sum of the coefficients of those like terms.

***Example* 1:**

(i) Add: $5x^2 - 7x + 3$, $-8x^2 + 2x - 5$ and $7x^2 - x - 2$

(ii) Simplify: $(2x^3 - 2x^2 - 2) - (2x^3 - 2x - 1) - (2x^3 - x^2 - x + 1)$

Solution:

(i) $(5x^2 - 7x + 3) + (-8x^2 + 2x - 5) + (7x^2 - x - 2) =$

$$= \underbrace{\left(5x^2 - 8x^2 + 7x^2\right) + \left(-7x + 2x - x\right) + \left(3 - 5 - 2\right)}_{collecting\ \ like\ \ terms}$$

$$= (5 - 8 + 7)x^2 + (-7 + 2 - 1)x + \left(3 - 5 - 2\right)$$

$$= \mathbf{4x^2 - 6x - 4}.$$

(ii) $(2x^3 - 2x^2 - 2) - (2x^3 - 2x - 1) - (2x^3 - x^2 - x + 1)$

$$= 2x^3 - 2x^2 - 2 - 2x^3 + 2x + 1 - 2x^3 + x^2 + x - 1$$

$$= (2x^3 - 2x^3 - 2x^3) + (-2x^2 + x^2) + (2x + x) + (-2 + 1 - 1)$$

$$= \mathbf{-2x^3 - x^2 + 3x - 2}.$$

***Example* 2:**

Multiply: (i) $(3x+4)(2x-1)$

(ii) $(x-2)(x^2+3x-1)$

Solution: (i) $(3x+4)(2x-1)$

$$= 6x^2 - 3x + 8x - 4$$
$$= 6x^2 + 5x - 4$$

(ii) $(x-2)(x^2+3x-1)$

$$= x^3 + 3x^2 - x - 2x^2 - 6x + 2$$
$$= x^3 + x^2 - 7x + 2$$

3.2 Algebraic Identities

An **algebraic identity** is an equality that holds true regardless of the values chosen for its variables. Since identities are true for all valid values of its variables, one side of the equality can be swapped for the other.

Square of a Sum / Difference

If a and b represent some algebraic expressions, then

$$\boldsymbol{(a + b)^2 = a^2 + 2ab + b^2}$$

$$\boldsymbol{(a - b)^2 = a^2 - 2ab + b^2}$$

Caution: Keep in mind that $(a + b)^2 \neq a^2 + b^2$

Difference of two Squares

If a and b represent some algebraic expressions, then

$$a^2 - b^2 = (a + b)(a - b)$$

***Example* 1:** Evaluate: (i) $(2x + 3)^2$ (ii) $(3m - 2n)^2$

Solution:

(i) $(2x + 3)^2 = (2x)^2 + 2(2x)(3) + 3^2$
$= 4x^2 + 12x + 9$

(ii) $(3m - 2n)^2 = (3m)^2 - 2(3m)(2n) + (2n)^2$
$= 9m^2 - 12mn + 4n^2$

***Example* 2:** **Find the product**

(i) $(4x - 5y)(4x + 5y)$

(ii) $(xy - 5y)(xy + 5y)$

(iii) $(x + 1)^2 - (x - 1)^2$

Solution:

(i) $(4x - 5y)(4x + 5y) = 16x^2 - 25y^2$

(ii) $(xy - 5y)(xy + 5y) = x^2y^2 - 25y^2$

(iii) $$\begin{aligned}(x+1)^2 - (x-1)^2 &= [(x+1) + (x-1)][(x+1) - (x-1)] \\ &= (2x)(2) \\ &= 4x\end{aligned}$$

***Example* 3:**

If $3a - b = 5$ and $ab = 6$ find $9a^2 + b^2$

Solution:

Given that $3a - b = 5$ and $ab = 6$

$\Rightarrow (3a - b)^2 = 9a^2 - 6ab + b^2$

$\Rightarrow 5^2 = 9a^2 - 6 \times 6 + b^2$

$$\Rightarrow \quad 25 = 9a^2 - 36 + b^2$$

$$\Rightarrow \quad 9a^2 + b^2 = 25 + 36$$

$$\Rightarrow \quad 9a^2 + b^2 = 61$$

3.3 Factorization of Expressions

Term by Term Division

If a, b and c represent some algebraic expressions, then

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

i.e, if a polynomial is divided by a monomial, then each term of the polynomial must be divided by the monomial.

***Example* 1: Divide:** (i) $\dfrac{2a^3b - 6a^2b^2}{2a^2b^2}$ (ii) $\dfrac{(2a^3b)(-6a^2b^2)}{2a^2b^2}$

Solution:

(i) $\dfrac{2a^3b - 6a^2b^2}{2a^2b^2} = \dfrac{2a^3b}{2a^2b^2} - \dfrac{6a^2b^2}{2a^2b^2} = \dfrac{a}{b} - 3$

(ii) $\dfrac{(2a^3b)(-6a^2b^2)}{2a^2b^2} = \dfrac{-12a^5b^3}{2a^2b^2} = -6a^3b$

Factoring a Monomial from a polynomial

Consider an expression of the form *ab + ac*. Clearly, a is a common factor for the terms *ab* and *ac*. Thus we may write:

$$ab + ac = a(b + c)$$

***Example* 2:** Factorize the expression $3x^2y^3z - 12x^3y^2$

Solution:
[Here consider the coefficients: 3 and -12. We know that 3 is the greatest common factor for these numbers. So we may factor 3 from them. From x^2 and x^3, we can factor x^2 from them. y^2 can be factored from y^3 and y^2. Since the variable z is not present in the second term, we cannot factor z.]

Thus, $3x^2y^3z - 12x^3y^2 = 3x^2y^2(yz - 4x)$

Factoring by Grouping

To factorize an expression with no common factor to all terms and with an even number of terms, we may use the method of grouping as explained below:

Consider the expression, $ac + ad + bc + bd$.

Here, there are no common factors to all the four terms. But, if we group the terms as ($ac + ad$) and ($bc + bd$), we can factor the terms a and b respectively from these groups.

Thus,

$ac + ad + bc + bd = (ac + ad) + (bc + bd)$ [*grouping*]

$= a(c + d) + b(c + d)$ [*factoring each group*]

$= (c + d)(a + b)$ [*factoring* (c + d) *from both terms*]

***Example* 3: Factorize the following expressions:**

(i) $5m^2n - 10mn^2 + mn$ (ii) $(2x+4)(x-3)-5(x-3)$

(iii) $3m^2 - 4mn + 6m - 8n$ (iv) $2x^2 + 3x - 2x - 3$

Solution

(i) $5m^2n - 10mn^2 + mn = mn(5m - 10n + 1)$

(ii) $(2x+4)(x-3)-5(x-3) = [(2x+4)-5](x-3) = (2x-1)(x-3)$

(iii) $3m^2 - 4mn + 6m - 8n = (3m^2 - 4mn) + (6m - 8n)$

$= m(3m - 4n) + 2(3m - 4n) = (3m - 4n)(m + 2)$

(iv) $2x^2 + 3x - 2x - 3 = (2x^2 + 3x) - (2x + 3)$ [*notice the sign of the last term*]

$= x(2x + 3) - (2x + 3) = (2x + 3)(x - 1)$

Factoring expressions of the form x^2+px+q

Consider the expressions of the form $x^2 + px + q$. Suppose that

$$x^2 + px + q = (x+a)(x+b)$$

Then, upon multiplying,

$$x^2 + px + q = (x+a)(x+b) = x^2 + (a+b)x + ab$$

Thus $q = ab$ and $p = a + b$, that is, the constant term is the product of two numbers, and the coefficient of x is the sum of these two numbers.

***Example* 4: Factorize the following expressions:**

(i) $x^2 + 5x + 6$ (ii) $x^2 - 5x + 6$

(iii) $x^2 - x - 6$ (iv) $x^2 + 10x + 24$

Solution

(i) $x^2 + 5x + 6 = (x+2)(x+3)$

(ii) $x^2 - 5x + 6 = (x-2)(x-3)$

(iii) $x^2 - x - 6 = (x-3)(x+2)$

(iv) $x^2 + 10x + 24 = (x+4)(x+6)$

Special Factorization

In this section, we discuss the factorization of expressions of the form
$a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$ and $a^2 - b^2$.

We use the following identities to factor these expressions:

a)	$\boldsymbol{a^2 + 2ab + b^2 = (a + b)^2}$	[Square of a sum]
b)	$\boldsymbol{a^2 - 2ab + b^2 = (a - b)^2}$	[Square of a difference]
c)	$\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$	[Difference of two squares]

***Example* 5: Factorize the following expressions:**

(i) $9x^2 + 12x + 4$ (ii) $4m^2 - 20mn + 25n^2$

(iii) $16x^2 - 9y^2$ (iv) $x^4 - 9y^2$

Solution

(i) $9x^2 + 12x + 4 = (3x + 2)^2$

(ii) $4m^2 - 20mn + 25n^2 = (2m - 5n)^2$

(iii) $16x^2 - 9y^2 = (4x + 3y)(4x - 3y)$

(iv) $x^4 - 9y^2 = (x^2 - 3y)(x^2 + 3y)$

CHAPTER 4 - RATIONAL EXPRESSIONS

A rational expression is an expression that can be written in the form $\frac{P}{Q}$ where P and Q are polynomials and Q $\neq$0.

$\frac{2}{3}$, $\frac{3y^3}{8}$, $\frac{-4p}{p^3+2p+1}$, $\frac{5x^2-3x+2}{3x+7}$ are different examples of rational expressions.

The **reciprocal** of a rational expression $\frac{P}{Q}$ is $\frac{Q}{P}$.

Rules for Simplifying Rational Expressions

1. A rational expression is said to be in the simplest form if its numerator and denominator do not have any common factors.
2. We may add, subtract, multiply and divide the rational numbers using the following rules:

- $\frac{P}{Q}+\frac{R}{S}=\frac{PS+QR}{QS}$
- $\frac{P}{Q}-\frac{R}{S}=\frac{PS-QR}{QS}$
- $\frac{P}{Q}\times\frac{R}{S}=\frac{PR}{QS}$
- $\frac{P}{Q}\div\frac{R}{S}=\frac{P}{Q}\times\frac{S}{R}=\frac{PR}{QS}$

***Example* 1:**

Express the rational expression $\frac{x^2-1}{x^2-2x+1}$ in the lowest form.

Solution

$$\frac{x^2-1}{x^2-2x+1}=\frac{(x-1)(x+1)}{(x-1)(x-1)}=\frac{(x+1)}{(x-1)}$$

***Example* 2:**

Express the rational expression $\frac{x^2+5x+6}{x^2-5x-14}$ in the lowest form.

Solution

$$\frac{x^2+5x+6}{x^2-5x-14}=\frac{(x+2)(x+3)}{(x+2)(x-7)}=\frac{(x+3)}{(x-7)}$$

***Example* 3:** Add: $\dfrac{3}{x-3}+\dfrac{2}{x+2}$

Solution

$$\frac{3}{x-3}+\frac{2}{x+2}=\frac{3(x+2)}{(x-3)(x+2)}+\frac{2(x-3)}{(x+2)(x-3)}$$

$$=\frac{3x+6}{(x-3)(x+2)}+\frac{2x-6}{(x+2)(x-3)}$$

$$=\frac{5x}{(x-3)(x+2)} \quad = \quad \frac{5x}{x^2-x-6}$$

***Example* 4:** Multiply: $\dfrac{x-1}{x+1}\cdot\dfrac{x+2}{x-2}$

Solution

$$\frac{x-1}{x+1}\cdot\frac{x+2}{x-2}=\frac{(x-1)(x+2)}{(x+1)(x-2)} \quad = \quad \frac{x^2+x-2}{x^2-x-2}$$

***Example* 5:** Divide: $\dfrac{x-1}{x+1}\div\dfrac{x+2}{x-2}$

Answer:

$$\frac{x-1}{x+1}\div\frac{x+2}{x-2} \quad = \quad \frac{x-1}{x+1}\cdot\frac{x-2}{x+2} \quad = \quad \frac{(x-1)(x-2)}{(x+1)(x+2)} \quad = \quad \frac{x^2-3x+2}{x^2+3x+2}$$

CHAPTER 5 – EQUATIONS

An **equation** is a statement that two algebraic expressions are equal.

Eg: $5y + 9 = 7$

$8z - 2y = 3y$

$x^2 - 5x = 2$

Solution of an Equation

Consider the equation $x - 5 = 7$. Here 'x' is the variable in the equation and our aim is to find out the value of 'x' that makes the equation true .Clearly we can say that $x = 12$.
By substituting $x = 12$ in above equation we will get $12 - 5 = 7$. So $x = 12$ is the solution of the above equation.
The values of the variable that make the equation true are called the **solutions** or **roots** of the equation and the process of finding the solutions is called **solving the equation** .

5.1 Linear Equations in One Variable

An equation involving only one variable with the highest power of the variable 1 is called a linear equation in one variable. The general form of linear equation in one variable is **a*x* + b = 0** , where *'x'* is the variable and '**a**' and '**b**' are constants .

Eg: $7x - 2 = 0$

$9 - x = 5$

$12 + b = 15 - 5b$

Solution of a linear equation: The value of the variable that makes the equation true is called its ***solution*** or ***root***. A linear equation in one variable has exactly one solution.

Given below are some examples of linear and non-linear equations.

Linear equations	Nonlinear equations
$6x - 5 = 9$	$x^2 + 3x = 9$
$y - 7 = 3y$	$\sqrt{x} + 7x = 9$
$\frac{1}{3}x - 9x = 8$	$\frac{3}{x} - 4x = 2$

***Example* 1 : Solve the following linear equations for *x*:**

(i) $x - a + b = 2a + 3b$

(ii) $\frac{x}{2} - \frac{x}{3} = \frac{x}{4} + \frac{1}{2}$

Solution: (i) We have,

$$x - a + b = 2a + 3b$$

$$\Rightarrow \quad x = 2a + 3b + a - b$$

Collecting the term of x on one side and other terms to the other side

$$\Rightarrow \quad x = 3a + 2b$$

$\therefore$ The required solution is $\boldsymbol{x = 3a + 2b}$**.**

(ii) Consider,

$$\frac{x}{2} - \frac{x}{3} = \frac{x}{4} + \frac{1}{2}$$

$$\frac{3x - 2x}{6} = \frac{2x + 4}{8}$$

$$\frac{x}{6} = \frac{x+2}{4}$$

$$4x = 6x + 12$$

$$4x - 6x = 12$$

$$-2x = 12$$

$$x = -6$$

$\therefore$ The required solution is $\boldsymbol{x = -6}$.

5. 2 Quadratic Equations

The standard form of a **quadratic equation** is $\boldsymbol{ax^2 + bx + c = 0}$, where a, b and c are real numbers with $a \neq 0$. The degree of a quadratic equation is 2.

Zero –Product Property				
AB = 0	***if and only if***	***A=0***	***or***	***B*=0**

1) Solving a quadratic Equation by factoring

***Example* 1:** Solve the equation $x^2 + 5x = 24$

Solution: Consider, $x^2 + 5x = 24$

$\Rightarrow \quad x^2 + 5x - 24 = 0$ — *Converting to the standard form.*

$\Rightarrow \quad (x - 3)(x + 8) = 0$ — *Factoring the quadratic expression.*

$\Rightarrow \quad x - 3 = 0 \quad$ or $\quad x + 8 = 0$

$\Rightarrow \quad x = 3 \quad$ or $\quad x = -8$

The solutions are $x = \mathbf{3}$ and $x = \mathbf{-8}$

Solving the quadratic equation when b = 0

***Example* 2:** Solve the quadratic equation $x^2 - 25 = 0$

Solution: We have, $x^2 - 25 = 0$

$$\Rightarrow \quad (x - 5)(x + 5) = 0$$

$$\Rightarrow \quad x - 5 = 0 \quad \text{or} \quad x + 5 = 0$$

$$\Rightarrow \quad x = 5 \quad \text{or} \quad x = -5$$

The solutions are **+5** and **-5.**

2) Solving the equation using quadratic formula

The Quadratic formula
The solutions of the quadratic equation $ax^2 + bx + c = 0$,where $a \neq 0$ are given by, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

***Example* 3**: Solve the equation $x^2 - 4x + 3 = 0$

Solution: Here we have, a = 1, b = -4 and c = 3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 12}}{2} \qquad = \frac{4 \pm \sqrt{4}}{2}$$

$$= \frac{4 \pm 2}{2} \qquad = \frac{4+2}{2} \text{ or } \frac{4-2}{2}$$

$$= \frac{6}{2} \text{ or } \frac{2}{2} \qquad = 3 \text{ or } 1$$

$\therefore$ The solutions are **3** and **1**

***Example* 4:** Solve the equation $3x^2 - 5x - 1 = 0$

Solution: Here we have, a = 3, b = -5 and c = -1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{25 + 12}}{6} \qquad = \frac{5 \pm \sqrt{37}}{6}$$

The solutions are $\frac{5+\sqrt{37}}{6}$ and $\frac{5-\sqrt{37}}{6}$

***Example* 5**: Solve the equation $x^2 + 2x + 2 = 0$

Solution: Here we have, a = 1, b = 2 and c = 2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2}$$

Since, $\sqrt{-4}$ is not real, there are no real solutions for the given equation.

Discriminant and the nature of roots

The value, $\boldsymbol{D = b^2 - 4ac}$ is called the **discriminant** of the quadratic equation $ax^2 + bx + c = 0$.

We can identify the nature of the roots of the equation using the discriminant as given below.

Case 1:	If $\boldsymbol{D = b^2 - 4ac > 0}$, then there are two distinct real roots for the equation.
Case 2:	If $\boldsymbol{D = b^2 - 4ac = 0}$, then there is only one real root for the equation.
Case 3:	If $\boldsymbol{D = b^2 - 4ac < 0}$, then there are no real roots for the equation.

***Example* 6: Evaluate the discriminant of the following quadratic equations and comment on their roots:**

1) $x^2 + 4x - 1 = 0$

Solution: Here a = 1, b = 4 and c = -1

Discriminant, $D = b^2 - 4ac = 4^2 - 4(1)(-1)$

$= 16 + 4 = \mathbf{20}$

Since $b^2 - 4ac = 20 > 0$, the equation has two distinct real roots.

2) $4x^2 - 12x + 9 = 0$

Solution: Here a = 4, b = -12 and c = 9

Discriminant, $D = b^2 - 4ac = (-12)^2 - 4(4)(9)$

$= 144 - 144 = \mathbf{0}$

Since $b^2 - 4ac = 0$, the equation has exactly one real root.

3) $x^2 + 2x = -2$

Solution: We have $x^2 + 2x + 2 = 0$

Here a = 1, b = 2 and c = 2

Discriminant, $D = b^2 - 4ac \quad = 2^2 - 4(1)(2)$
$= 4 - 8 = \mathbf{-4}$

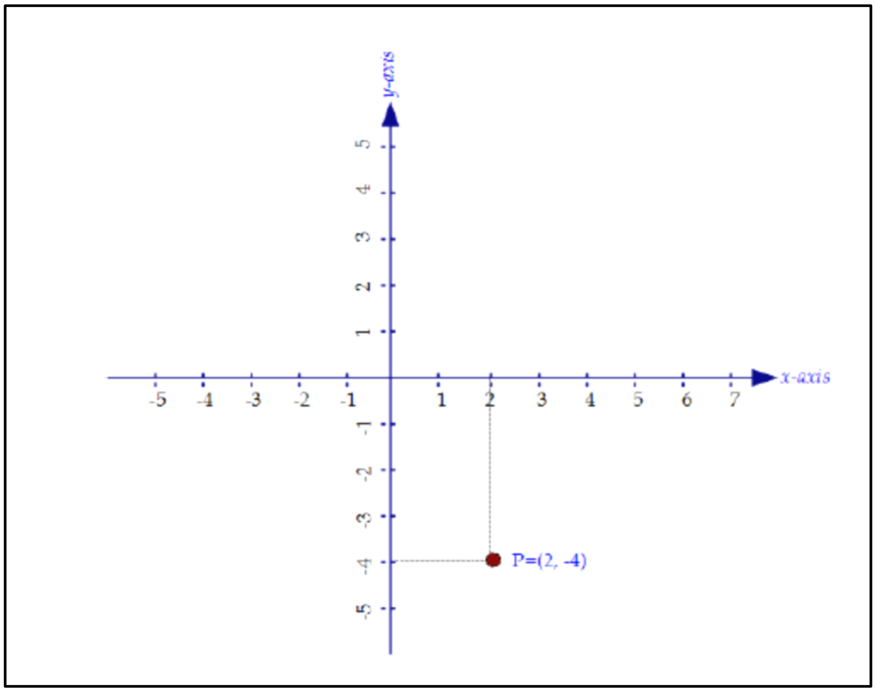
Since $b^2 - 4ac < 0$, the equation has no real roots.

CHAPTER 6 - COORDINATE GEOMETRY

6.1 Introduction to Coordinate Geometry

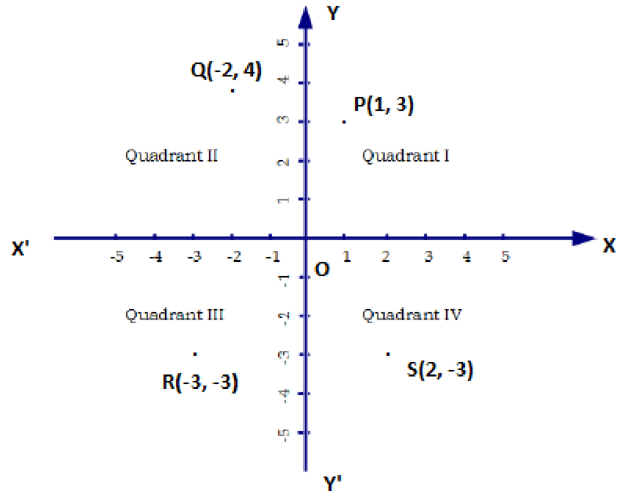
A plane containing two perpendicular number lines, one horizontal and the other vertical, meeting each other at 0 is called a **Cartesian plane** or **coordinate plane.** We use these two lines to locate the exact position of each point in the plane. The horizontal number line is called the **x–axis** and the vertical number line is called the **y – axis.** The x – axis is denoted by X'OX and the y – axis is denoted by Y'OY as give in the figure below.

Each point P on the plane is associated to an ordered pair of values in the form (x, y) where x and y are the perpendicular distances drawn from the point to the y – axis and x – axis respectively. The first value in the pair is called the **x coordinate** or the **abscissa** and the second value is called the **y coordinate** or the **ordinate.** The x coordinate of any point on the y axis is 0 and the y coordinate of any point on the x axis is 0.



The point where the two axes meet each other is called the **origin.** The origin is always denoted by O(0, 0).

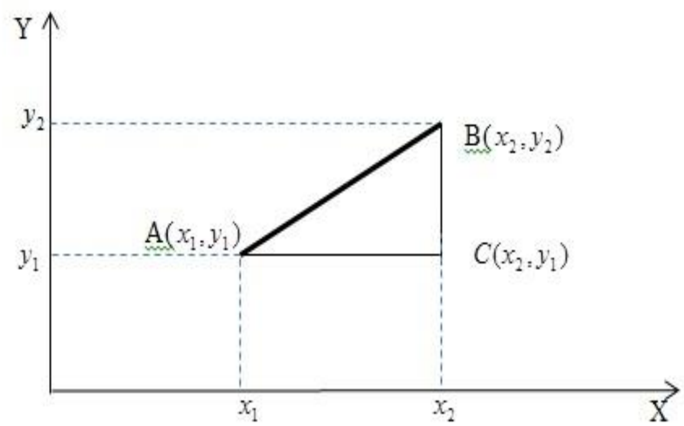
The two axes divide a Cartesian plane into four regions called the **quadrants** namely, the first (I), second (II), third (III) and the fourth (IV) quadrants. Each point in the first quadrant will be of the form (+, +), in II quadrant, it is (-, +), in III it is (-, -) and in IV quadrant it is (+, -).



Thus the point P(1, 3) is in I quadrant, Q(-2, 4) in the II quadrant, R(-3, -3) in the III quadrant and S(2, -3) in the IV quadrant.

6.2 DISTANCE AND MIDPOINT FORMULA

Let us find the distance between two points $A(x_1, y_1)$ and B (x_2, y_2) in the plane.



The distance between the points $A(x_1, y_1)$ and B (x_2, y_2) is given by

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note: The distance between a point A(x,y) and the Origin is $\sqrt{x^2 + y^2}$.

***Example* 1:** Find the distance between P(-2, 3) and Q(1, -3).

Solution: Here $x_1 = -2$, $y_1 = 3$, $x_2 = 1$ and $y_2 = -3$.
Then the distance is given by,

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-(-2))^2 + (-3-3)^2} \\ &= \sqrt{9+36} \\ &= 3\sqrt{5} \quad \text{units} \end{aligned}$$

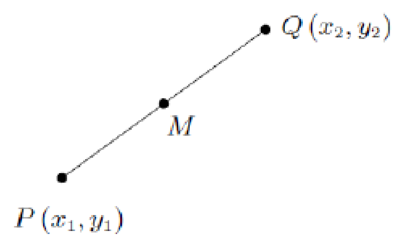
***Example* 2:** Find the distance between the points (4,3) and the origin.

Solution: Here $d = \sqrt{x^2 + y^2} = \sqrt{4^2 + 3^2}$
$= \sqrt{25}$ = **5 units**

Midpoint Formula

Consider two points P(x_1, y_1) and Q(x_2, y_2) in a plane. Then the midpoint M of the line segment is the point on PQ such that PM = QM

Clearly, the coordinates of M are given by,
M = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



***Example* 3:** Find the midpoint of the line segment connecting P(-2, 3) and Q(1, -3).

Solution: Here $x_1 = -2$, $y_1 = 3$, $x_2 = 1$ and $y_2 = -3$.

Then the midpoint M is given by,

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{(-2)+1}{2}, \frac{3+(-3)}{2}\right) \\ &= \left(-\frac{1}{2}, \frac{0}{2}\right) = \left(-\frac{1}{2}, 0\right) \end{aligned}$$

Thus the midpoint is $\left(-\frac{1}{2}, 0\right)$.

QUESTION PAPER PATTERN

The question paper consists of 30 multiple choice questions carrying 1 mark each. A student must answer a minimum of 18 questions to pass the exam. The weightages of questions per each section are given in the following table:

No.	Topics	# of questions / Marks
1	Real Number system	4
2	Exponents	3
3	Algebraic Expressions	4
4	Algebraic Identities	3
5	Factorization	4
6	Rational expressions	2
7	Linear Equations	3
8	Quadratic Equations	3
9	Coordinate Geometry	2
10	Distance & Midpoint	2
TOTAL		**30**